Digital Processing Laboratory
Laboratorio de Procesamiento Digital

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Real Time Digital Signal Processing

UTN-FRBA
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www.electron.frba.utn.edu.ar/dplab

Multirate DSP

Decimation

Interpolation

Polyphase Filters

IFIR Filters

CIC Filters

Outline

- ***Introduction***
- Sample Rate Conversion
- Decimation
  - Two - Stage
- Interpolation
  - Two - Stage
- Combining Decimation and Interpolation
- Polyphase Filters
- Application examples

Introduction

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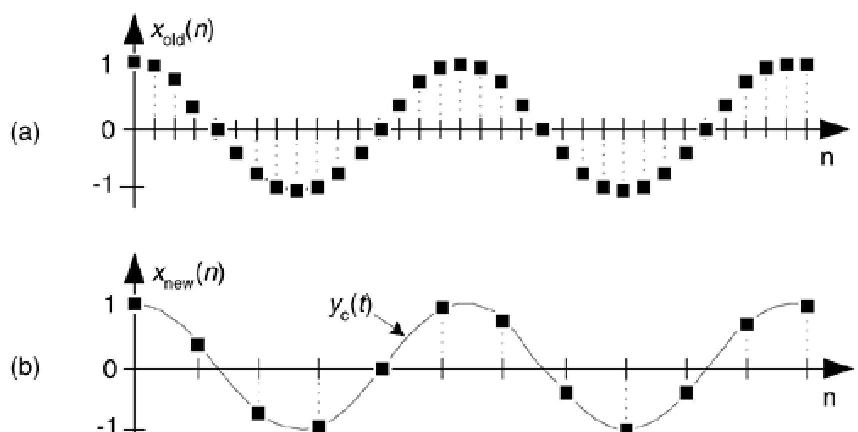
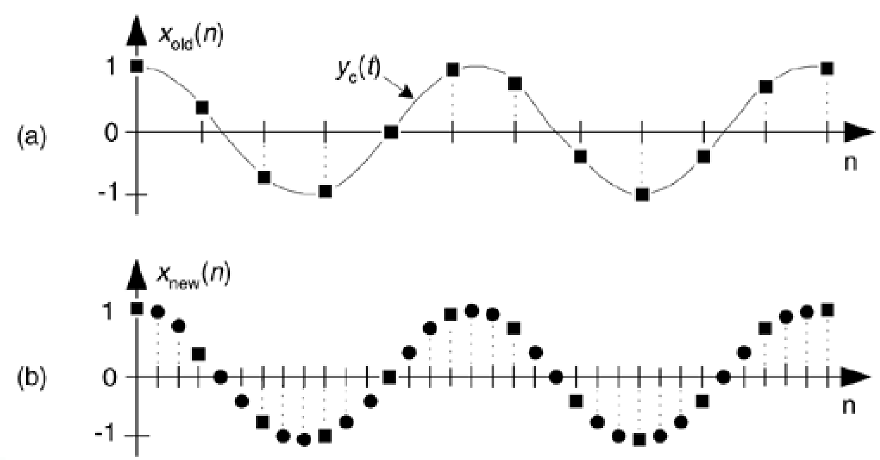
Sample Rate Conversion

- Consider the process where a continuous signal x(t) has been sampled at a rate of $f_{s,old} = 1/T_{old}$, and the discrete samples are $x_{old}(n) = x(nT_{old})$.
- Rate conversion is necessary when we need $x_{new}(n) = x(nT_{new})$, and direct sampling of the continuous $x(t)$ at the rate of $f_{s,new} = 1/T_{new}$ is not possible.
- How do we obtain $x_{new}(n)$ directly from *xold(n)*?
  - One possibility is to digital-to-analog (D/A) convert the $x_{old}(n)$ sequence to regenerate the continuous $x(t)$ and then A/D convert $x(t)$ at a sampling rate of $f_{s,new}$ to obtain $x_{new}(n)$.
  - Due to the spectral distortions induced by D/A followed by A/D conversion, this technique limits our effective dynamic range and is typically avoided in practice.
  - Fortunately, accurate all-digital sample rate conversion schemes have been developed, as we shall see.

Sample Rate Conversion

- Sampling rate changing come in two flavors:
  - Rate decreases - $\downarrow M$
    - Its typically called ***decimation***
    - Signal amplitude doesn't changes
  - Rate increases - $\uparrow L$
    - Its typically called ***interpolation***
    - It has amplitude loss
- It’s not time invariant.





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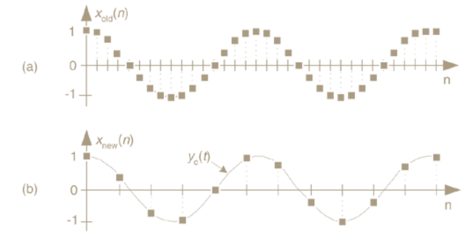
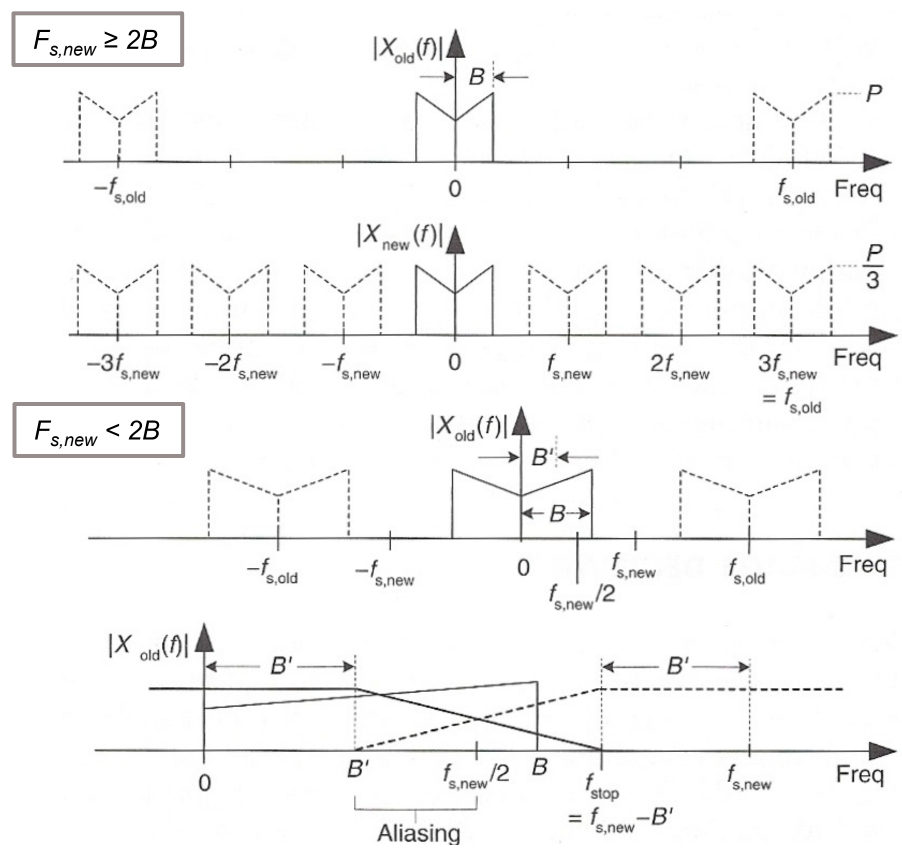
Decimation

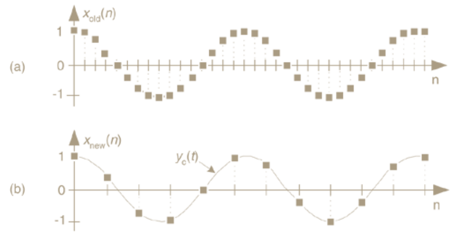


- Decimation is the two-step processes of ***low-pass filtering*** followed by an operation know as ***downsampling***.
- We are using an alternate time index variable ***m***, rather than ***n***, to remind us that the time period between the ***$x_{new}(m)$*** samples is different from the time period between the ***$x_{old}(n)$*** samples.

$$f_{s,new} = \frac{f_{s,old}}{M} \quad , \quad x_{new}(m) = x_{old}(Mn)$$



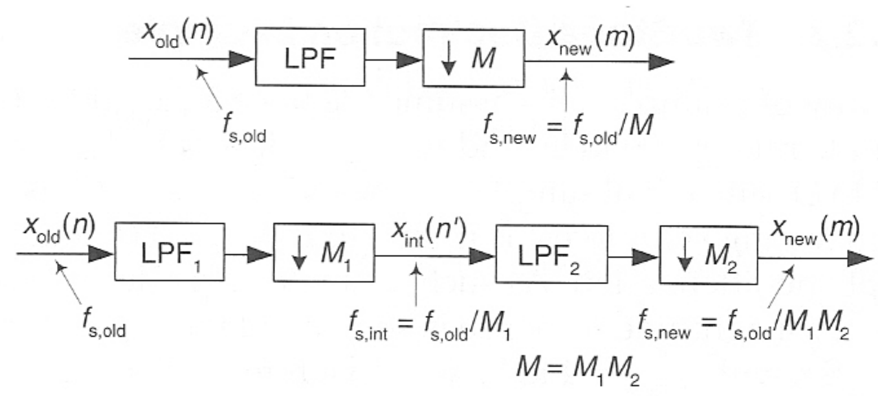
Decimation



- If the original signal has a bandwidth B, and we're interested in retaining only the band B', the signal above B' must be lowpass filtered, with full attenuation in the stopband beginning at f_{stop}, before the decimation process is performed.
- In practice, the nonrecursive ***FIR*** filter structure is the prevailing choice for ***decimation filters*** due to its ***linear phase response***.
- It's not necessary compute filter output samples that are discarded. Digital ***Polyphase Filters*** avoid these computational inefficiencies.

Two-Stage Decimation

- When the desired decimation factor M is large (M>20) there is an important feature of the filter/decimation process to keep in mind.
- The system in the figure are called multirate systems because there are two or more different data sample rate within a single system.



What should be the values of D_1 and D_2 to minimize the number of taps in lowpass filters LPF_1 and LPF_2?

$$M_{1,opt} \approx 2M \frac{1-\sqrt{MF/(2-F)}}{2-F(M+1)} \quad , \quad F = \frac{f_{stop} - B'}{f_{stop}}$$

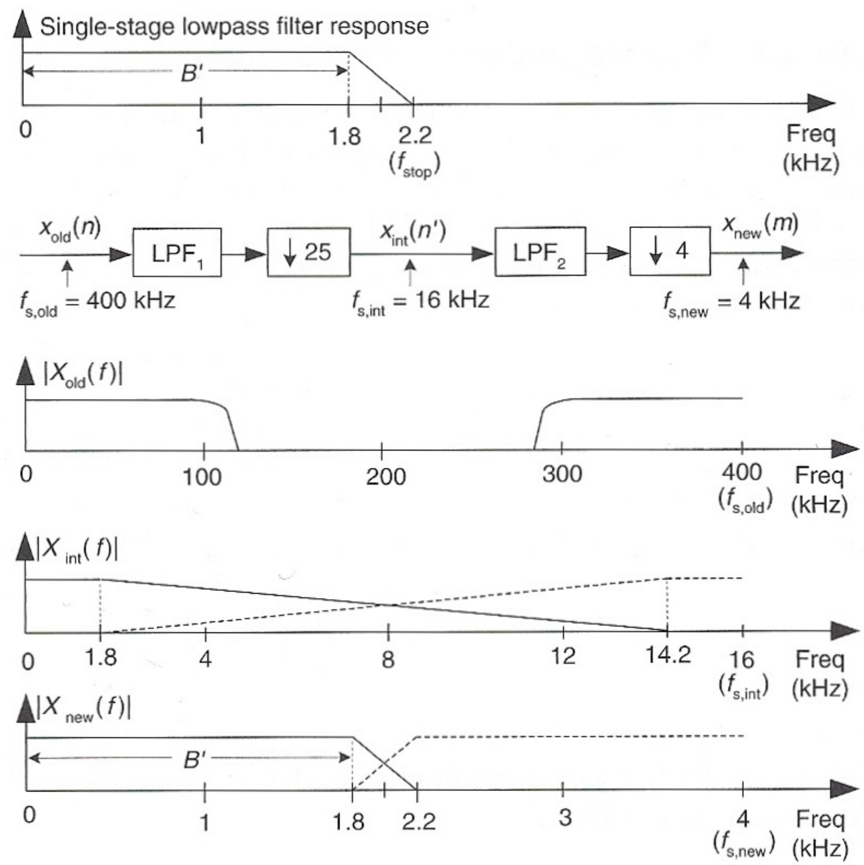
Two-Stage Decimation Example

$f_{s,old}$ = 400KHz , B > 100KHz
$f_{s,new}$ = 4KHz , B' = 1.8KHz
60 dB stopband attenuation

$M = f_{s,old} / f_{s,new} = 100$
$f_{stop} = f_{s,new} - B' = 2.2\text{KHz}$
$N_{FIR} = \text{Atten} / [22(f_{stop} - f_{pass})/f_s]$
$= 60 / [22*(2.2-1.8)/400]$
$= 2727$

F = (2200-1800)/2200 = 0.182
$M_{1,opt} \approx 26.4 \rightarrow M_1 = 25$, $M_2 = 4$

$N_{LPF1} = 88$ $N_{LPF2} = 109$



1 stage: N_{FIR}xM multiplications
2 stages: $N_{LPF1}xM_1+N_{LPF2}xM_2$ multiplications

Two-Stage Decimation

- It's always to our benefit to decimate in ***order from the largest to the smallest*** factor. ($M_1 > M_2$)
- It's advantageous to consider setting the $\boldsymbol{M_1}$ and $\boldsymbol{M_2}$ decimation factors equal to ***integer powers of two*** because we can use computationally efficient half-band filters for lowpass filters.
- If the dual filter system required a passband peak-peak of R dB, then ***both filters*** must be designer to have a ***passband peak-peak ≤ R/2 dB***.
- The number of multiplications needed to compute each $x_{new}(m)$ is much larger than N_{total}. ***Polyphase filters*** only requires $\boldsymbol{N_{total}}$ ***multiplications*** per $x_{new}(m)$.

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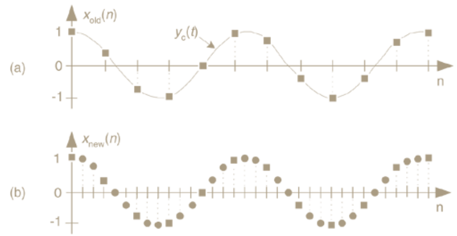
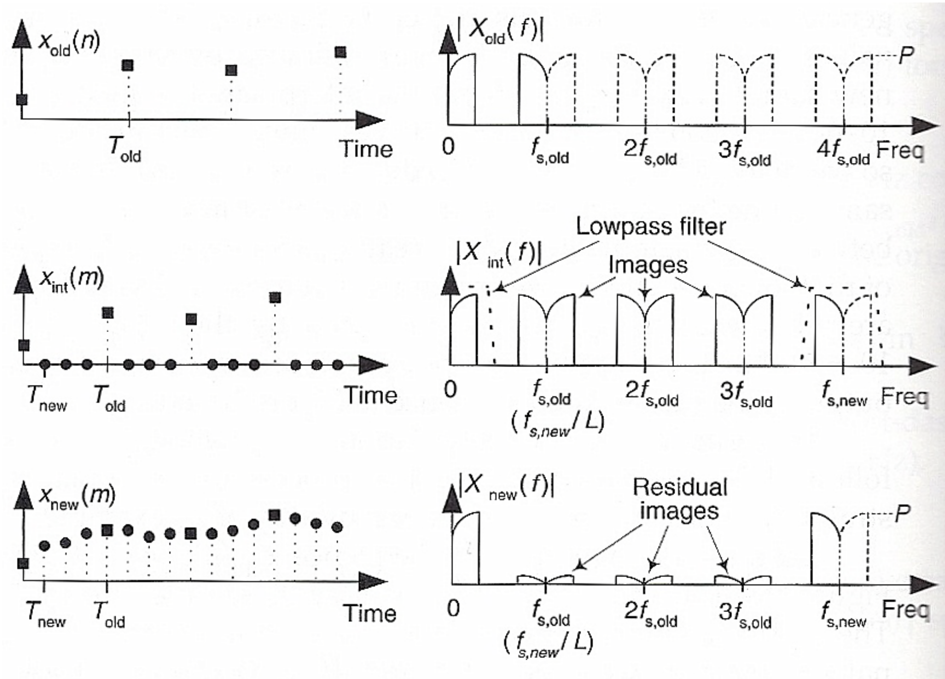
Interpolation

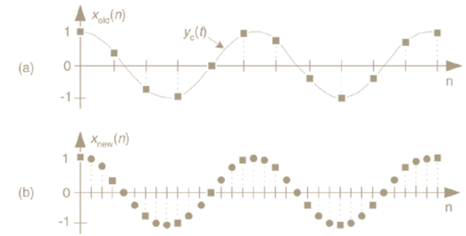


- Interpolation is the two-step processes of an operation know as ***upsampling*** followed by ***low-pass filtering***.
- To increase a given $f_{s,old}$ by an integer factor of L, we must to insert ***L-1 zero-valued samples*** behind each sample in $x_{old}(n)$.

$$f_{s,new} = L \cdot f_{s,old}$$



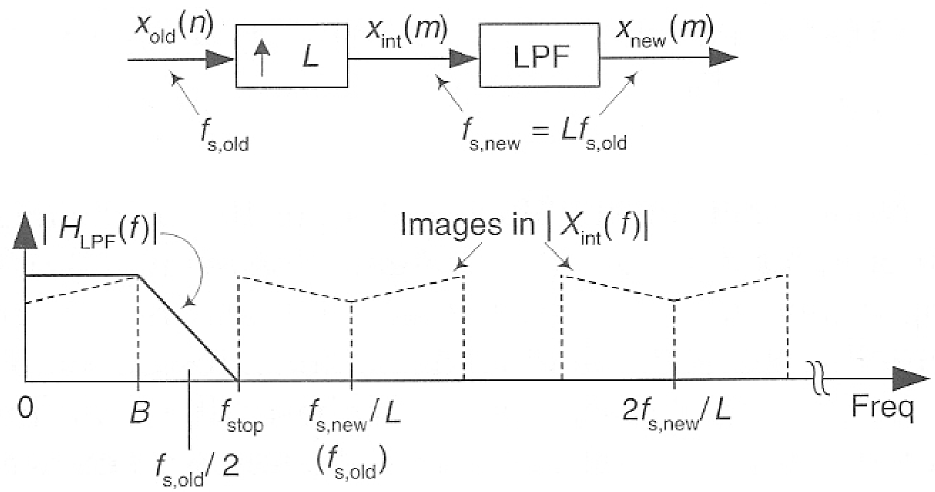
Interpolation

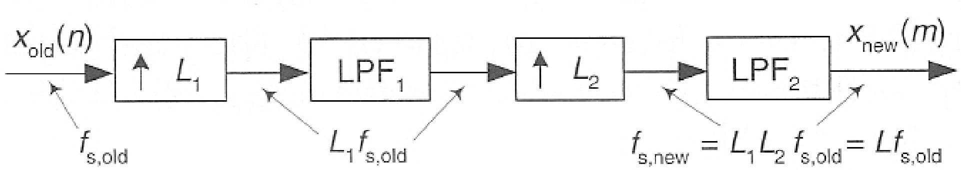


- We ***can't implement*** an ***ideal lowpass filter***, $x_{new}(m)$ will not be an exact interpolation of $x_{old}(n)$. The error manifests itself as the ***residual images*** within $X_{new}(m)$.
- We can only approximate an ideal lowpass interpolation filter. The ***greater the stopband attenuation, the more accurate the interpolation.***
- Interpolation process, because of the zero-valued samples, has an inherent ***amplitude loss factor of L***. Thus to achieve unity gain between sequences $x_{old}(n)$ and $x_{new}(m)$, the ***interpolation filter must have a gain of L***.

Two-Stage Interpolation

- When the desired interpolation factor L is large (L>20) there is an important feature of the filter/interpolate process to keep in mind.
- The system in the figure are called multirate systems because there are two or more different data sample rate within a single system.





$$L_{2,opt} \approx 2L\frac{1-\sqrt{LF/(2-F)}}{2-F(L+1)} \quad , \quad F = \frac{f_{stop}-B}{f_{stop}}$$

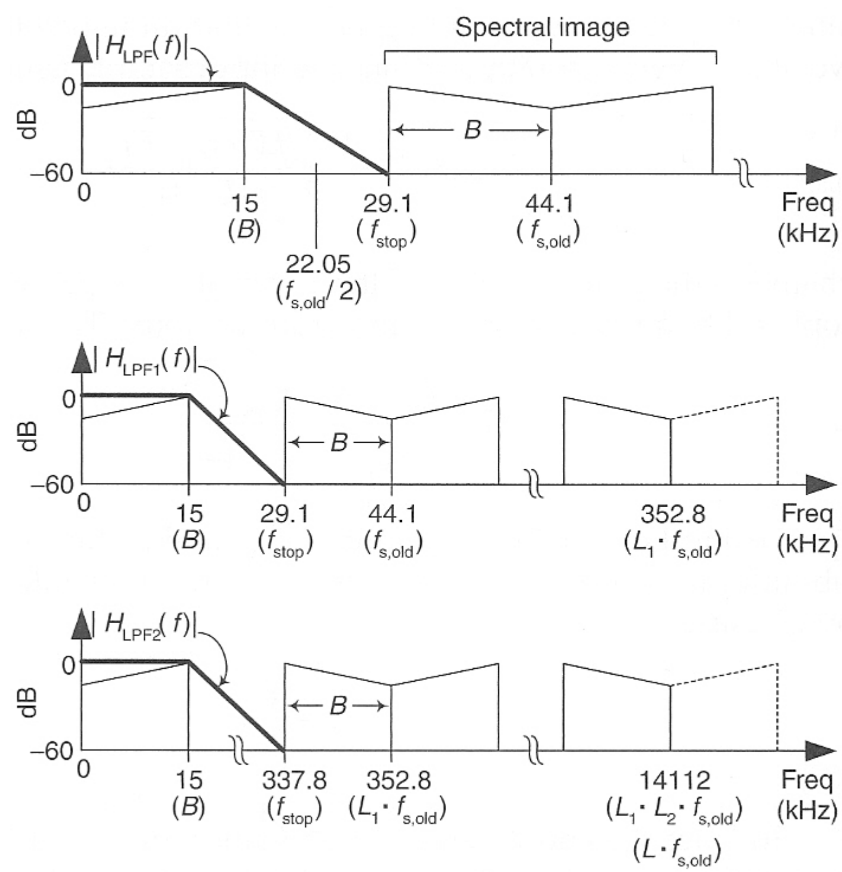
Two-Stage Interpolation Example

$f_{s,old}$ = 44.1KHz , B = 15KHz
$f_{s,new}$ = 14.112MHz
60 dB stopband attenuation

L = $f_{s,new}$ / $f_{s,old}$ = 320
f_{stop} = $f_{s,old}$ – B = 29.1KHz
N_{FIR} = Atten / [22(f_{stop} - f_{pass})/f_s]
= 60 / [22*(29.1-15)/14112]
= 2730

F = (29.1-15)/29.1 = 0.4845
$L_{2,opt}$ ≈ 37.98 → L_2= 40 , L_1= 8

N_{LPF1}= 68 N_{LPF2} = 119



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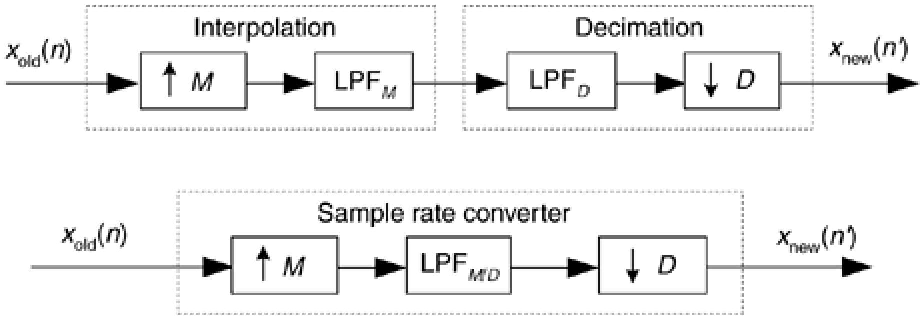
Combining Decimation and Interpolation

- We can implement sample rate conversion by any rational fraction L/M with interpolation by an integer factor of L followed by decimation by an integer factor of M.
- For hardware interpolator/decimators, we strive to implement designs optimizing the conflicting goals of high performance (minimum aliasing), simple architecture, high data throughput speed, and low power.



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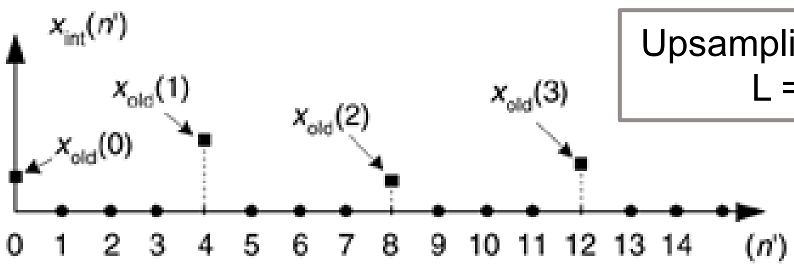
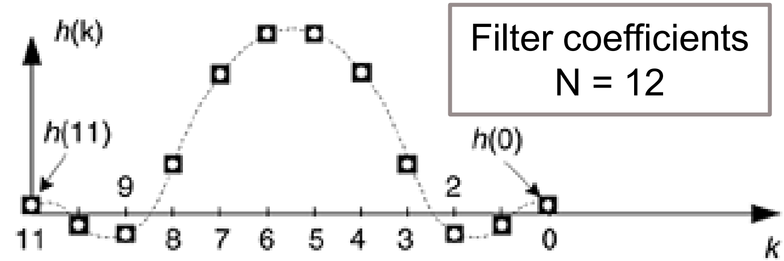
Polyphase Filters

- Interpolation
  - eliminate all multiply by zero operations.
- Decimation
  - avoid the wasteful computation of filter output samples that are subsequently discarded.



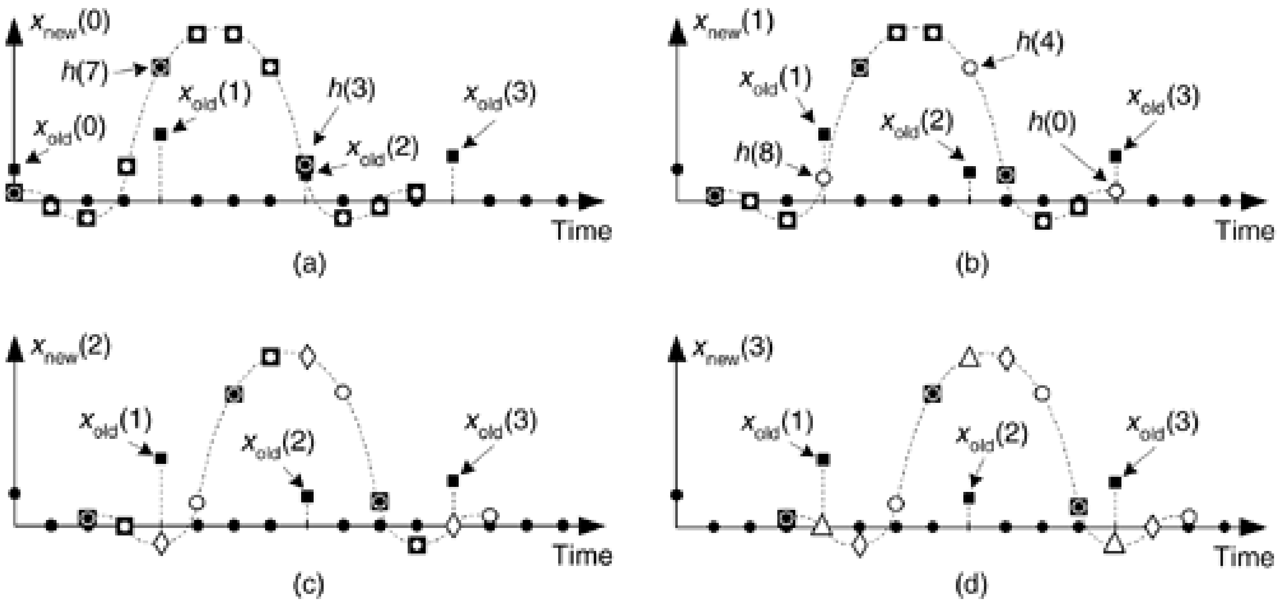
Upsampling signal
L = 4



Filter coefficients
N = 12

"For my money, the development of polyphase filters arguably resides in the stratosphere of brilliant DSP innovations, along with the radix-2 FFT algorithm and the Parks-McClellan FIR filter design algorithm" *R. G. Lyons*

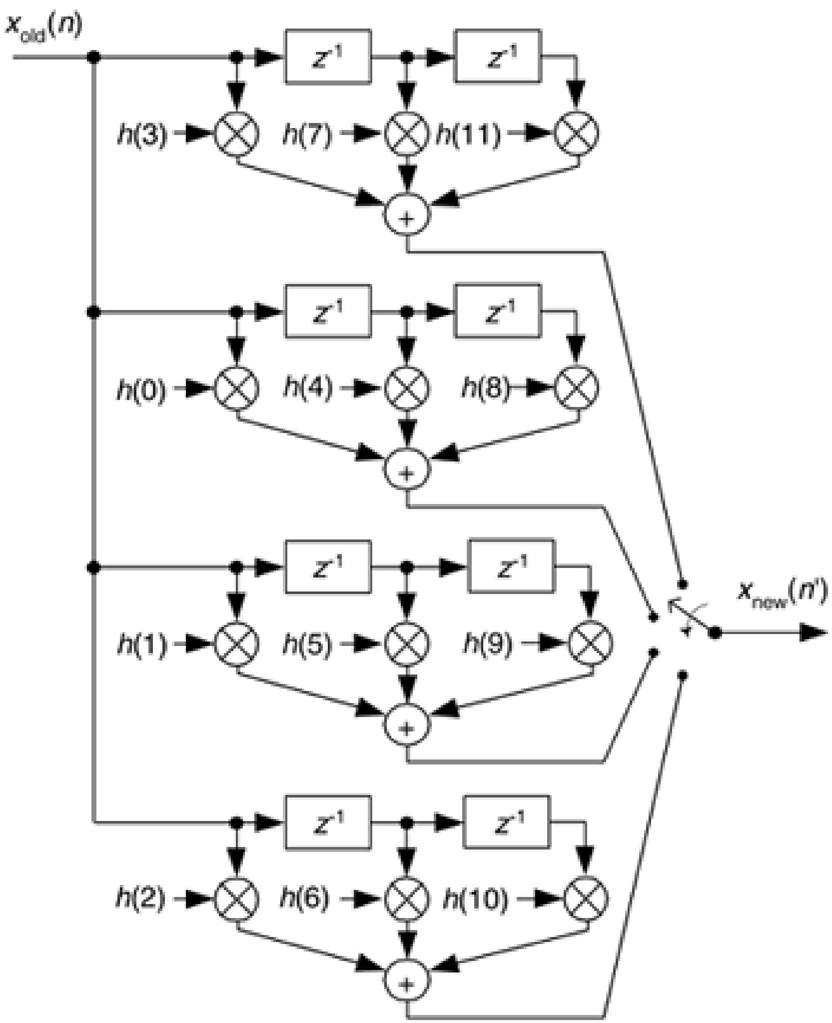
Polyphase Interpolation

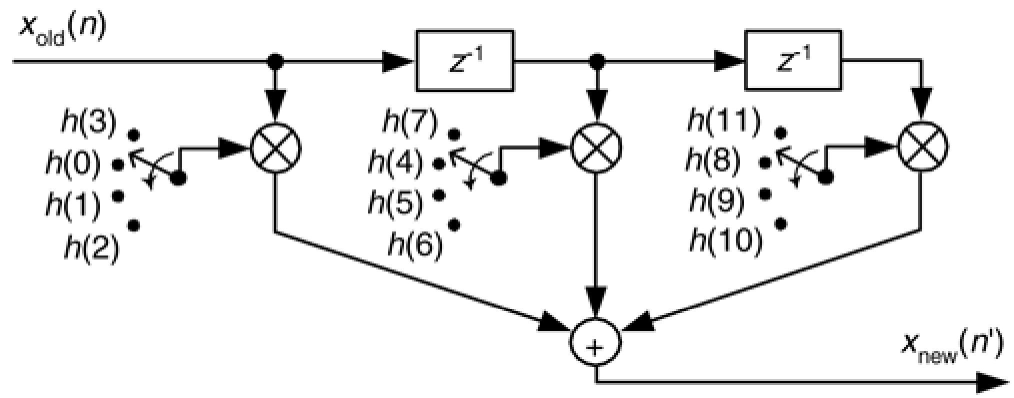


$$
\begin{aligned}
x_{new}(0) &= h(3)x_{old}(2) + h(7)x_{old}(1) + h(11)x_{old}(0) && \leftarrow \text{uses the } \boxtimes \text{ coefficients}\\
x_{new}(1) &= h(0)x_{old}(3) + h(4)x_{old}(2) + h(8)x_{old}(1) && \leftarrow \text{uses the o coefficients}\\
x_{new}(2) &= h(1)x_{old}(3) + h(5)x_{old}(2) + h(9)x_{old}(1) && \leftarrow \text{uses the } \Diamond \text{ coefficients}\\
x_{new}(3) &= h(2)x_{old}(3) + h(6)x_{old}(2) + h(10)x_{old}(1) && \leftarrow \text{uses the } \Delta \text{ coefficient}\\
x_{new}(4) &= h(3)x_{old}(3) + h(7)x_{old}(2) + h(11)x_{old}(1) && \leftarrow \text{uses the } \boxtimes \text{ coefficients}\\
x_{new}(5) &= h(0)x_{old}(4) + h(4)x_{old}(3) + h(8)x_{old}(2) && \leftarrow \text{uses the o coefficients}\\
x_{new}(6) &= h(1)x_{old}(4) + h(5)x_{old}(3) + h(9)x_{old}(2) && \leftarrow \text{uses the } \Diamond \text{ coefficients}\\
x_{new}(7) &= h(2)x_{old}(4) + h(6)x_{old}(3) + h(10)x_{old}(2) && \leftarrow \text{uses the } \Delta \text{ coefficients}
\end{aligned}
$$

Slide the impulse response to the right

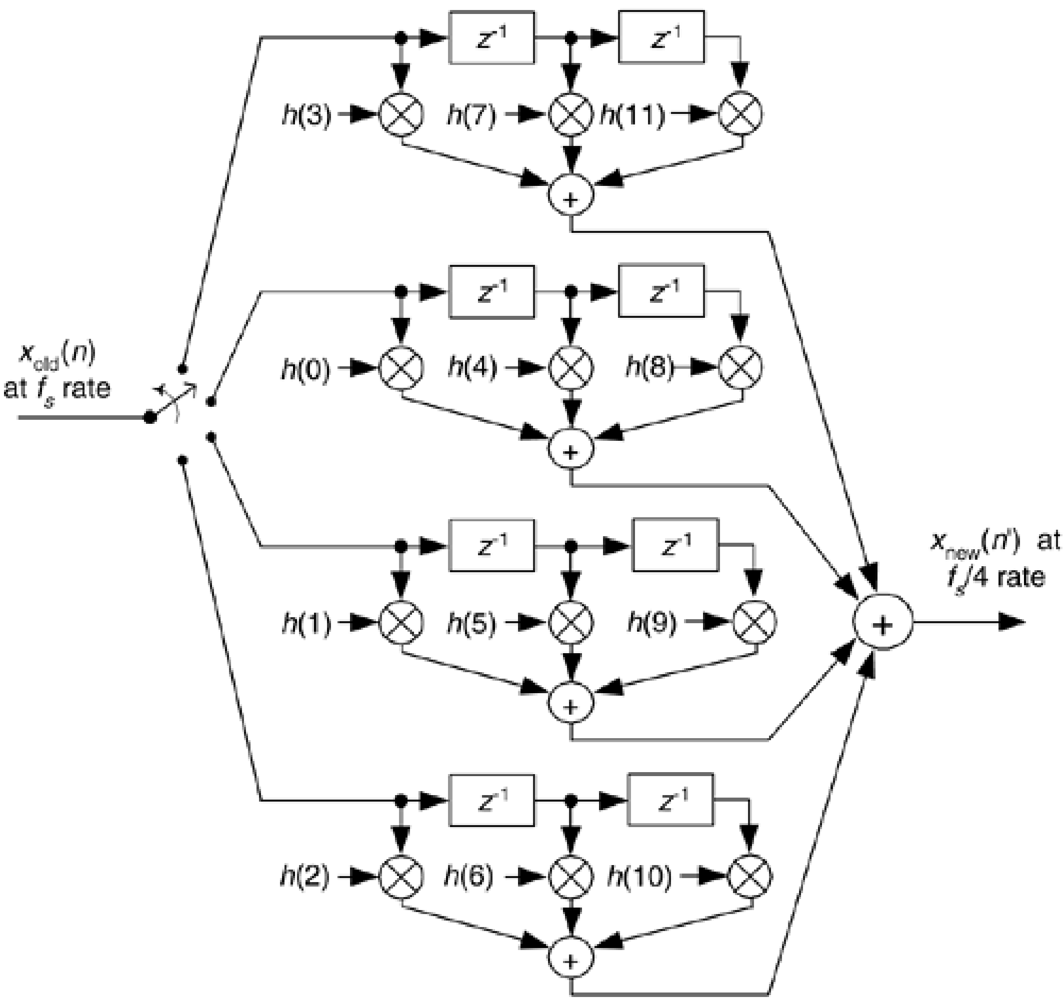
Polyphase Interpolation





- Don't created $x_{int}(m)$
- Don't multiply by zero
- L subfilters
- N/L – 1 delay elements
- N is chosen to be an integer multiple of L

Polyphase Decimation



- Don't created $x_{int}(m)$
- Don't performed unnecessary computations
- M subfilters
- N/M – 1 delay elements
- N is chosen to be an integer multiple of M

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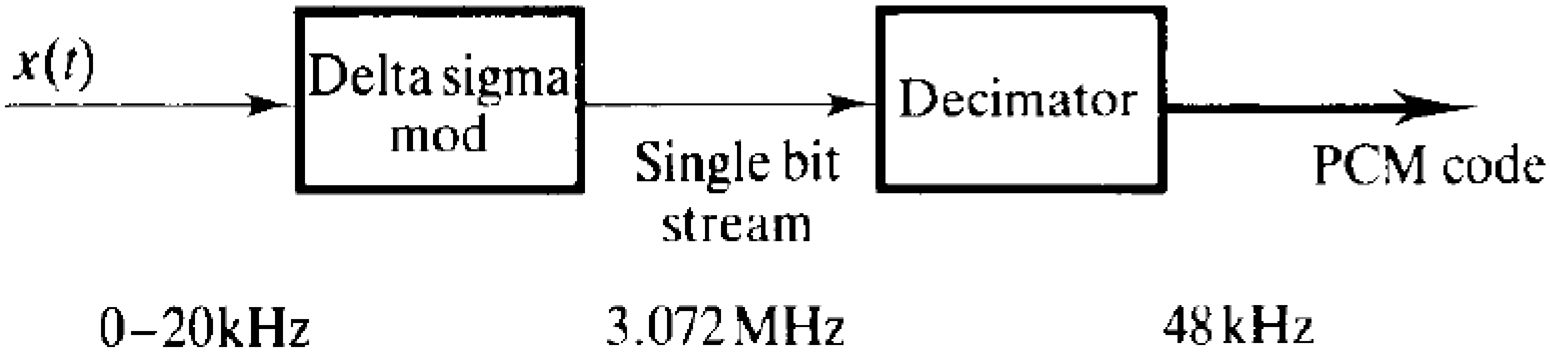
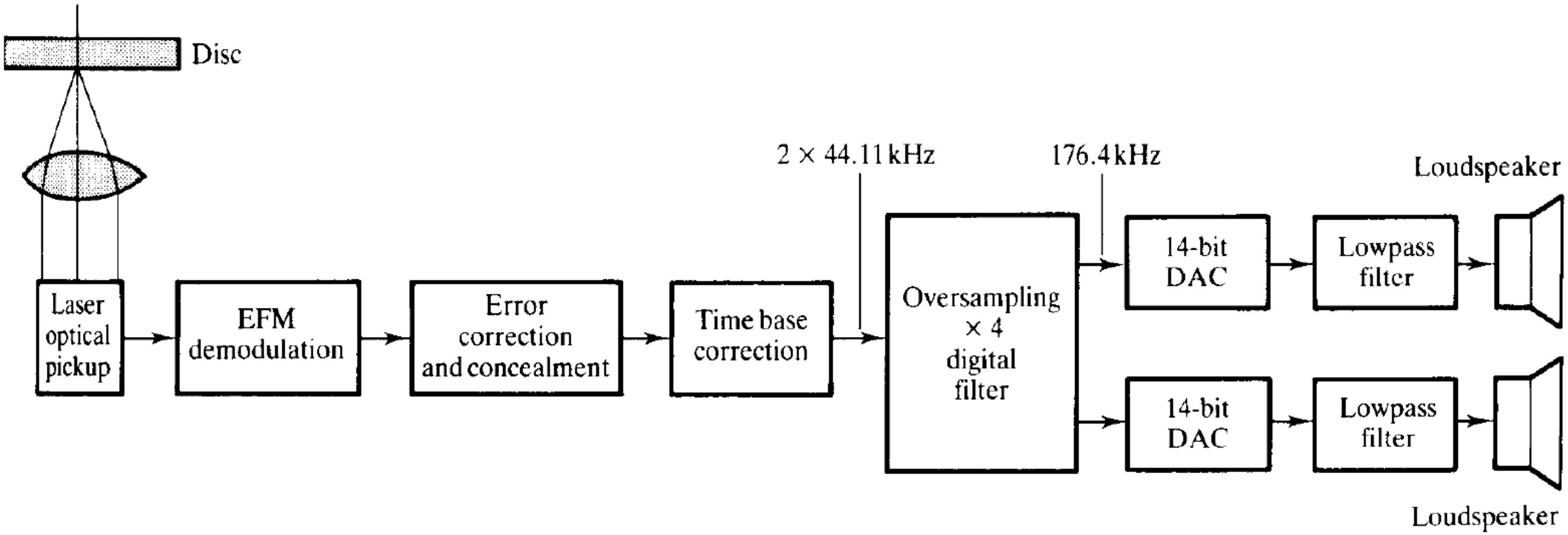
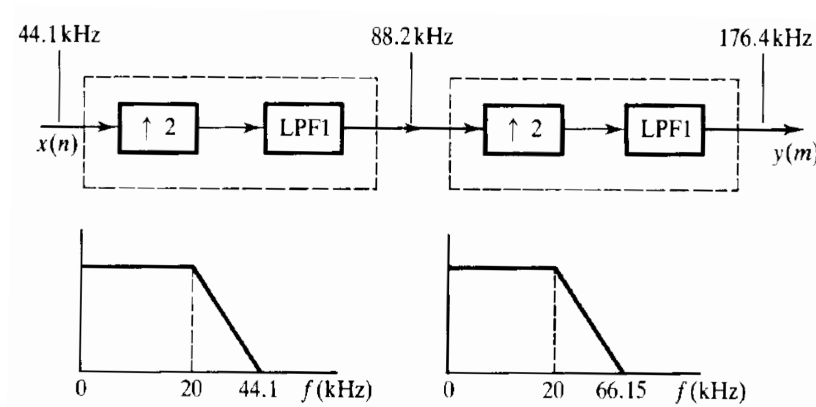
High quality A/D conversion for digital audio

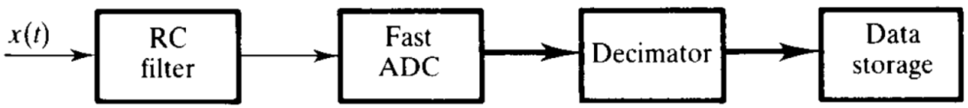


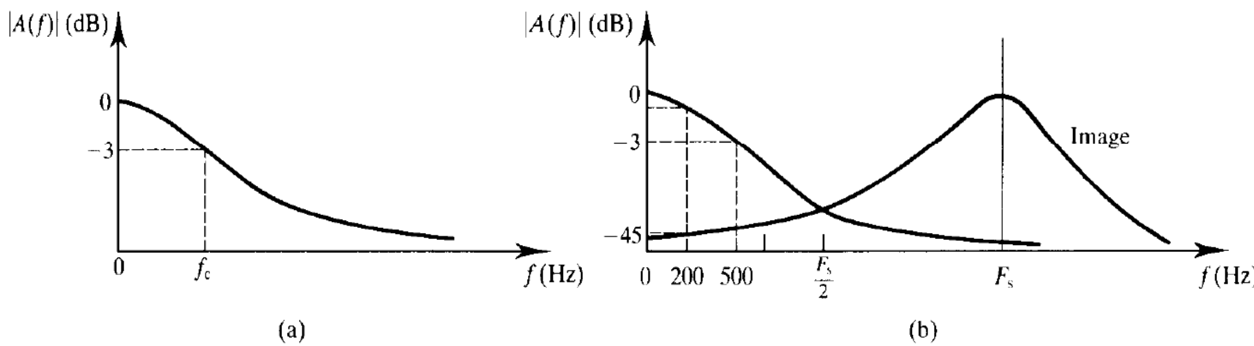
Efficient D/A conversion in compact hi-fi system

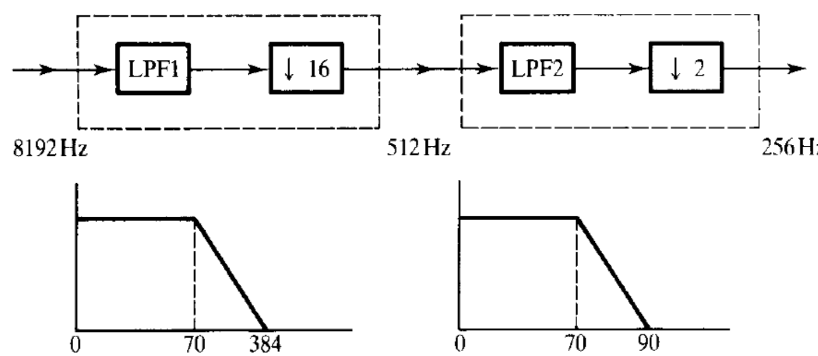




Application in the acquisition of high quality data



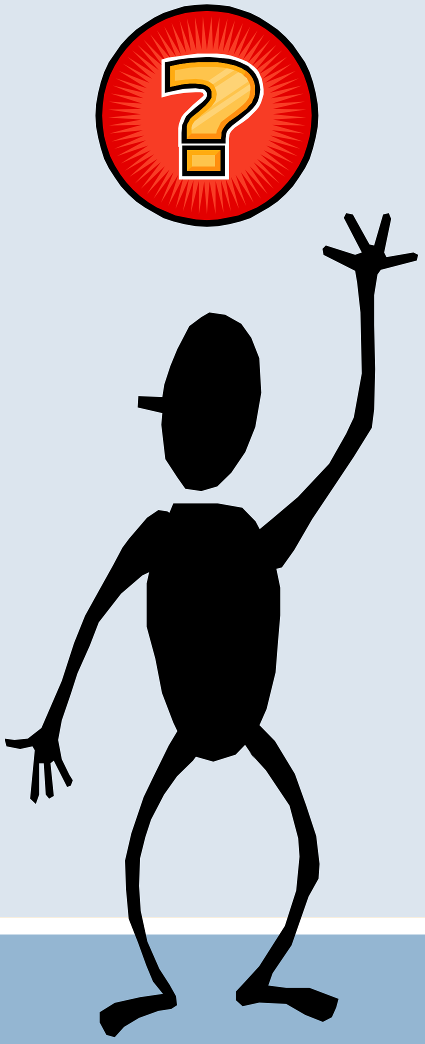




Recommended bibliography

- FJ Harris, Multirate Signal Processing for Communication Systems. First Edition. Prentice Hall 2004.
- RG Lyons, Understanding Digital Signal Processing. Third Edition. Prentice Hall 2010.
  - Ch10: Sample Rate Conversion.
- EC Ifeachor, BW Jervis. Digital Signal Processing. A Practical approach. Second Edition. Prentice Hall 2002.
  - Ch9: Multirate digital signal processing

NOTE: Many images used in this presentation were extracted from the recommended bibliography.



Questions?

Thank you!